

پاسخنامه تشریحی

۱ - گزینه ۲ می‌دانیم: $\cot a - \tan a = 2 \cot 2a$

$$\tan \frac{x}{2} - \cot \frac{x}{2} = -(\cot \frac{x}{2} - \tan \frac{x}{2}) = -2 \cot x = -2\left(\frac{1}{\tan x}\right) = -2\left(\frac{3}{4}\right) = -\frac{3}{2}$$

۲ - گزینه ۴

$$\sin \alpha = 2 \sin \frac{\alpha}{2} \cos \frac{\alpha}{2}, \quad 1 + \cos \alpha = 2 \cos^2 \frac{\alpha}{2}$$

$$\cot \alpha + \frac{1}{\sin \alpha} = \frac{\cos \alpha}{\sin \alpha} + \frac{1}{\sin \alpha} = \frac{1 + \cos \alpha}{\sin \alpha} = \frac{2 \cos^2 \frac{\alpha}{2}}{2 \sin \frac{\alpha}{2} \cos \frac{\alpha}{2}} = \cot \frac{\alpha}{2}$$

۳ - گزینه ۳

$$\cos 165^\circ = \cos(180^\circ - 15^\circ) = -\cos 15^\circ, \quad \cos 105^\circ = \cos(90^\circ + 15^\circ) = -\sin 15^\circ$$

$$\Rightarrow \cos 165^\circ \cdot \cos 105^\circ = \cos 15^\circ \sin 15^\circ \Rightarrow \frac{1}{2} \sin 30^\circ = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$$

۴ - گزینه ۱

$$\begin{aligned} 1 + \cos u &= 2 \cos^2 \frac{u}{2} \\ \sin u &= 2 \sin \frac{u}{2} \cos \frac{u}{2} \end{aligned}$$

$$\frac{\sin \alpha}{1 + \cos \alpha} = \frac{1}{2} \rightarrow \frac{2 \sin \frac{\alpha}{2} \cos \frac{\alpha}{2}}{2 \cos^2 \frac{\alpha}{2}} = \tan \frac{\alpha}{2} = \frac{1}{2}$$

$$\tan\left(\frac{\pi}{2} + \frac{\alpha}{2}\right) = -\cot \frac{\alpha}{2} = \frac{-1}{\tan \frac{\alpha}{2}} = \frac{-1}{\frac{1}{2}} = -2$$

۵ - گزینه ۲

$$\cos 2a = \cos^2 a - \sin^2 a \quad \text{می‌دانیم:}$$

$$\tan \frac{2\pi}{3} \sin\left(\frac{2\pi}{3} - x\right) = 1 \Rightarrow \tan\left(\pi - \frac{\pi}{3}\right) \sin\left(\frac{2\pi}{3} - x\right) = 1$$

$$\Rightarrow -\tan \frac{\pi}{3} (-\cos x) = 1 \Rightarrow \sqrt{3} \cos x = 1 \Rightarrow \cos x = \frac{1}{\sqrt{3}}$$

$$\cos 2x = 2 \cos^2 x - 1 = 2 \times \frac{1}{3} - 1 = \frac{-1}{3}$$

$$\sin \alpha \cdot \cos \alpha = \frac{1}{2} \sin 2\alpha, \quad \sin \alpha - \cos \alpha = \sqrt{2} \sin\left(\alpha - \frac{\pi}{4}\right) \quad \text{۶ - گزینه ۳ می‌دانیم:}$$

$$\frac{1}{\sin 15^\circ} - \frac{1}{\cos 15^\circ} = \frac{\cos 15^\circ - \sin 15^\circ}{\sin 15^\circ \cos 15^\circ} = \frac{-\sqrt{2} \sin(15^\circ - 45^\circ)}{\frac{1}{2} \sin 30^\circ} = \frac{-\sqrt{2} \sin(-30^\circ)}{\frac{1}{2} \times \frac{1}{2}} = 2\sqrt{2}$$

۷ - گزینه ۴

$$\cos(a+b) = \cos a \cdot \cos b - \sin a \cdot \sin b$$

$$\sin(a-b) = \sin a \cdot \cos b - \cos a \cdot \sin b$$

$$\begin{aligned} 2 \cos\left(\frac{\pi}{4} + a\right) \cdot \sin\left(\frac{\pi}{4} - a\right) &= 2\left(\frac{\sqrt{2}}{2} \cos a - \frac{\sqrt{2}}{2} \sin a\right) \cdot \left(\frac{\sqrt{2}}{2} \cos a - \frac{\sqrt{2}}{2} \sin a\right) = 2 \times \frac{\sqrt{2}}{2} (\cos a - \sin a) \times \frac{\sqrt{2}}{2} (\cos a - \sin a) \\ &= (\cos a - \sin a)^2 = \cos^2 a + \sin^2 a - 2 \sin a \cdot \cos a = 1 - \sin 2a \end{aligned}$$

۸ - گزینه ۳

$$\cos\left(\frac{2\pi}{3} - \alpha\right) = \cos\left(2\frac{\pi}{3} - \alpha\right) = -\sin \alpha \quad \text{و} \quad 2 \cos^2 \alpha = 1 + \cos 2\alpha \quad \text{می‌دانیم}$$

$$2 \cos^2\left(\frac{2\pi}{3} - x\right) = 1 + \cos\left(\frac{2\pi}{3} - 2x\right)$$

$$2 \cos^2\left(\frac{\sqrt{3}\pi}{4} - x\right) - \cos^2 x (1 + \tan^2 x) = 1 + \cos\left(\frac{\sqrt{3}\pi}{2} - 2x\right) - \cos^2 x \times \frac{1}{\cos^2 x} = \cos\left(\frac{\sqrt{3}\pi}{2} - 2x\right) = -\sin 2x$$

۹ - گزینه ۴

$$\text{می‌دانیم } \tan \alpha + \tan \beta = \frac{\sin \alpha}{\cos \alpha} + \frac{\sin \beta}{\cos \beta} = \frac{\sin \alpha \cdot \cos \beta + \cos \alpha \cdot \sin \beta}{\cos \alpha \cdot \cos \beta} = \frac{\sin(\alpha + \beta)}{\cos \alpha \cos \beta}$$

$$\begin{aligned} \cos 50^\circ (\tan 70^\circ + \tan 10^\circ) &= \cos 50^\circ \times \frac{\sin 80^\circ}{\cos 70^\circ \cos 10^\circ} = \frac{\cos 50^\circ \cos 10^\circ}{\cos 70^\circ \cos 10^\circ} \\ &= \frac{\cos 50^\circ}{\cos 70^\circ} = \frac{\sin 40^\circ}{\sin 20^\circ} = \frac{2 \sin 20^\circ \cos 20^\circ}{\sin 20^\circ} = 2 \cos 20^\circ \end{aligned}$$

توجه: اگر $\alpha + \beta = 90^\circ$ آنگاه $\sin \alpha = \cos \beta$

$$10 - \text{گزینه ۱ می‌دانیم } \sin \alpha \cos \alpha = \frac{1}{2} \sin 2\alpha \quad \cos\left(\frac{\pi}{2} - \alpha\right) = \sin \alpha$$

$$\Lambda \cos a \cos b \cos\left(\frac{\pi}{2} - a\right) \cos\left(\frac{\pi}{2} - b\right) = \Lambda \cos a \cos b \sin a \sin b$$

$$= 2(\sin a \cos a)(\sin b \cos b) = 2 \sin 2a \sin 2b$$

$$\xrightarrow{a+b=\frac{\pi}{2}} 2 \sin 2a \sin 2\left(\frac{\pi}{2} - a\right) = 2 \sin 2a \sin\left(\frac{\pi}{2} - 2a\right) = 2 \sin 2a \cos 2a = \sin 4a$$

۱۱ - گزینه ۴

$$\text{می‌دانیم } 1 + \tan^2 \theta = \frac{1}{\cos^2 \theta}, \quad 1 + \cot^2 \theta = \frac{1}{\sin^2 \theta}, \quad \sin \theta \cos \theta = \frac{1}{2} \sin 2\theta$$

$$\frac{(1 + \tan^2 \theta)(1 + \cot^2 \theta)}{(1 - \sin^2 \theta) - \cos^2 \theta} = \frac{\frac{1}{\cos^2 \theta} \times \frac{1}{\sin^2 \theta}}{\cos^2 \theta - \cos^2 \theta} = \frac{1}{\underbrace{\cos^2 \theta (1 - \cos^2 \theta)}_{\sin^2 \theta}}$$

$$= \frac{1}{(\sin \theta \cos \theta)^2} = \frac{1}{\left(\frac{1}{2} \sin 2\theta\right)^2} = \frac{4}{\sin^2 2\theta} = 4 \sin^{-2} 2\theta$$